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On Nearly πPS Sampling Scheme

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Summary

A nearly πPS sampling scheme has been suggested. For the proposed sampling scheme, Yates and Grundy form of variance estimator takes non-negative values always. On comparing the efficiency of the proposed nearly πPS sampling scheme with other well-known πPS sampling schemes for sample size greater than two, empirically it has been observed that the performace of the proposed sampling scheme is highly satisfactory.

Key Works : Efficiency, Inclusion probability, Nonnegativity of variance estimator, Sampling scheme, variance estimator.

Introduction

All sampling schemes which provide more chance to large units as compared to smaller units would provide better estimator of population total or mean than those provided by equal probability sampling schemes. A sampling scheme in which inclusion probabilities are proportional to size of the units is known as πPS sampling scheme. For πPS sampling scheme, Horvitz-Thompson estimator is used. For efficient use of Horvitz-Thompson estimator the desirable properties are

(1) Units are selected with inclusion probabilities π_i = npi

(2) $\pi_{ij} > 0$ for all i, j ; for ensuring the entimability of the variance.

(3) $\pi_i \pi_j - \pi_{ij} \geq 0$for all i, j ; for the nonnegativity of the variance estimator.

(4) the sample size n is fixed.

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None of the πPS sampling schemes available so far, for any sample size n, is entirely satisfactory from the above four points. Sunter [3] suggested that if a sampling scheme satisfies the requirements 2 to 4 and the first requirement is approximately satisfied, that may be considered satisfactory. In this paper, a sampling scheme for sample size n has been suggested which is nearly a πPS sampling scheme and satisfies the requirements 2 to 4 as given above.

2. *Proposed Nearly πPS Sampling Scheme*

Suppose the population consists of N distinct and identifiable units ($U_1, U_2, \ldots, U_N$) and a sample of size n is desired to be drawn. Let Y_i and X_i denote respectively the value of the character under study and the auxiliary character for the i-th unit of the population ($i = 1, 2, \ldots, N$). Let X_i be known for allo i and $X = \Sigma X_i$. Set $P_i = X_i/X$. Before proposing the nearly πPS sampling scheme, we consider the following sampling scheme.

Sampling Scheme-1:

For the given population of N units, calculate

$$K_1 = \frac{1}{\sum_{i=1}^{N} \frac{P_i}{1-P_i}}, \quad K_r = \frac{1}{\sum_{i=1}^{N} \frac{P_i}{1-(1+K_1+\ldots+K_{r-1})P_i}}$$

Then select a sample of n units by varying probabilities with replacement with the probability for the i-th unit at different draws as under:

First draw: P_i

Second draw: $\frac{K_1 P_i}{1-P_i}$

Third draw: $\frac{K_2 P_i}{1-(1+K_1)P_i}$

l-th draw: $\frac{K_{l-1} P_i}{1-(1+K_1+\ldots+K_{l-2})P_i}$

n-th draw: $\frac{K_{n-1} P_i}{1-(1+K_1+\ldots+K_{n-2})P_i}$

Before obtaining the inclusion probabilities for this sampling scheme, a Lemma is presented which will be useful in proving an important property of the scheme.

LEMMA:

$$\text{Set } A_t = \left[1 - \frac{K_t P_i}{1 - \left(1 + \sum_{q=1}^{t-1} K_q\right) P_i} - \frac{K_t P_j}{1 - \left(1 + \sum_{q=1}^{t-1} K_q\right) P_j} \right]$$

where

$$K_t = \frac{1}{\sum^{N} \frac{P_i}{1 - (1 + K_1 + \ldots + K_{q-1})\, p_i}} \qquad t, q = 1, 2, \ldots, n-1$$

Then

A_t, t = 1, 2, . . ., n−1, will always assume positive values.

Proof: Noting the fact that $\frac{K_t P_i}{1 - \left(1 + \sum_{q=1}^{t-1} K_q\right) P_i}$ and $\frac{K_t P_j}{1 - \left(1 + \sum_{q=1}^{t-1} K_q\right) P_j}$ are respectively the selection probabilities for the i-th and j-th units at the (t+1)th draw and some of such probabilities for all the units is one, it is obvious that A_t, t= 1, 2, ..., n−1 will always be non-negative.

The inclusion probabilities for individual and pairwise units for sampling scheme-1 are given by the following theorems.

Theorem-1 : For the sampling scheme-1, the inclusion probability for the i-th unit is given by

$$\pi_i = (1 + K_1 + \ldots + K_{n-1}) P_i$$

Proof: A unit can be included in the sample if it is selected in any of the n steps (draws).

Thus,

π_i = Probability of selecting i-th unit in at least one of the n draws.

= 1 – Probability of not selecting i-th unit at any of the n draws.

$$= 1 - (1-P_i)\left(1 - \frac{K_1 P_i}{1-P_i}\right)\left[1 - \frac{K_2 P_i}{1-(1+K_1)P_i}\right]\cdots$$

$$\cdots\left[1 - \frac{K_{n-1} P_i}{1-(1+K_1+\ldots+K_{n-2})P_i}\right]$$

$$= 1 - (1-P_i)\frac{1-(1+K_1)P_i}{1-P_i}\left[\frac{1-(1+K_1+K_2)P_i}{1-(1+K_1)P_i}\right]\cdots$$

$$\cdots\left[\frac{1-(1+K_1+\ldots+K_{n-1})P_i}{1-(1+K_1+\ldots+K_{n-2})P_i}\right]$$

$$\pi_i = (1+K_1+\ldots+K_{n-1})P_i$$

Hence the Theorem.

Theorem–2 : For the sampling scheme 1, the pairwise π_{ij} is given by

$$\pi_{ij} = (1+K_1+\ldots+K_{n-1})(P_i+P_j) - 1 + (1-P_i-P_j)$$

$$\left[1 - \frac{K_1 P_i}{1-P_i} - \frac{K_1 P_j}{1-P_j}\right]\cdots\left[1 - \frac{K_{n-1} P_i}{1-(1+K_1+\ldots+K_{n-2})P_i} - \frac{K_{n-1} P_j}{1-(1+K_1+\ldots+K_{n-2})P_j}\right]$$

Proof: π_{ij} = Probability of including the (i,j)- th pair of units in the sample.

= 1 – Probability of atleast not selecting one of the i-th or j-th units.

$$= 1 - P(\bar{I} \cup \bar{J})$$

$$= 1 - \{P(\bar{I}) + P(\bar{J}) - P(\bar{I}\bar{J})\}$$

Where $P(\bar{i})$ = Probability of not selecting i-th unit in the sample.

$$= 1 - (1 + K_1 + \ldots + K_{n-1}) P_i$$

and $P(\bar{i}\,\bar{j})$ = Probability of not selecting (i, j)-th pair of units in the sample.

$$= (1 - P_i - P_j)\left[1 - \frac{K_1 P_i}{1 - P_i} - \frac{K_1 P_j}{1 - P_j}\right] \ldots$$

$$\ldots \left[1 - \frac{K_{n-1} P_i}{1 - (1 + K_1 + \ldots + K_{n-2}) P_i} - \frac{K_{n-1} P_i}{1 - (1 + K_1 + \ldots + K_{n-2}) P_i}\right]$$

Substituting the values of $P(\bar{i})$, $P(\bar{j})$ and $P(\bar{i}\,\bar{j})$, we get

$$\pi_{ij} = (1 + K_1 + \ldots + K_{n-1})(P_i + P_j) - 1 + (1 - P_i - P_j)$$

$$\left(1 - \frac{K_1 P_i}{1 - P_i} - \frac{K_1 P_j}{1 - P_j}\right) \ldots \left[1 - \frac{K_{n-1} P_i}{1 - (1 + K_1 + \ldots + K_{n-2}) P_i} - \frac{K_{n-1} P_j}{1 - (1 + K_1 + \ldots + K_{n-2}) P_j}\right]$$

Hence the Theorem.

Following important result also holds.

Theorem-3 : For the sampling scheme-1, $\pi_i \pi_j - \pi_{ij} \geq 0$ for all $i \neq j = 1, 2, \ldots, N$.

Proof: It is known that

$$\pi_i = [1 - P(\bar{i})] \text{ and } \pi_{ij} = 1 - [P(\bar{i}) + P(\bar{j}) - P(\bar{i}\,\bar{j})]$$

so

$$\pi_i \pi_j = [1 - P(\bar{i})][1 - P(\bar{j})]$$

$$= 1 - P(\bar{i}) - P(\bar{j}) + P(\bar{i}) P(\bar{j})$$

Thus,

$$\pi_i \pi_j - \pi_{ij} = P(\bar{i}) P(\bar{j}) - P(\bar{i}\,\bar{j})$$

$$= [1-(1+K_1+\ldots+K_{n-1})P_i]\ [1-(K_1+\ldots+K_{n-1})P_j]$$

$$-(1-P_i-P_j)\prod_{t=1}^{n-1}\left[1-\frac{K_t P_i}{1-\left(1+\sum_{q=1}^{t-1}K_q\right)P_i}-\frac{K_t P_j}{1-\left(1+\sum_{q=1}^{t-1}K_q\right)P_j}\right]$$

Denoting

$$A_t = \left[1-\frac{K_t P_i}{1-\left(1+\sum_{q=1}^{t-1}K_q\right)P_i}-\frac{K_t P_j}{1-\left(1+\sum_{q=1}^{t-1}K_q\right)P_j}\right]$$

and $\quad m = 1+K_1+\ldots+K_{n-1}$

we get

$$\pi_i\pi_j-\pi_{ij} = (1-mP_i)(1-mP_j)-(1-P_i-P_j)\prod_{t=1}^{n-1}A_t$$

Expressing $1-P_i-P_j$ as $(1-P_i)(1-P_j)-P_iP_j$ we get

$$\pi_i\pi_j-\pi_{ij} = (1-mP_i)(1-mP_j)+P_iP_j\prod_{t=1}^{n-1}A_t$$

$$-[1-(1+K_1)(P_i+P_j)+(1+2K_1)P_iP_j]\prod_{t=2}^{n-1}A_t$$

similarly expressing $1-(1+K_1)(P_i+P_j)+(1+2K_1)P_iP_j$ as

$[1-(1+K_1)P_i][1-(1+K_1)P_j]-K_1^2P_iP_j$, we get

$$\pi_i\pi_j-\pi_{ij} = (1-mP_i)(1-mP_j)+P_iP_j\prod_{t=1}^{n-1}A_t$$

$$+K_1^2P_iP_j\prod_{t=2}^{n-1}A_t-\Big[1-(1+K_1+K_2)(P_i+P_j)+$$

$$(1+K_1^2+2K_1+2K_2+2K_1K_2)P_iP_j\Big]\prod_{t=3}^{n-1}A_t$$

With the same procedure, we get

$$\pi_i\pi_j - \pi_{ij} = K_{n-1}^2 P_iP_j + K_{n-2}^2 P_iP_jA_{n-1} + K_{n-3}^2 P_iP_j \prod_{t=n-2}^{n-1} A_t$$

$$+ \ldots + K_2^2 P_iP_j \prod_{t=3}^{n-1} A_t + K_1^2 P_iP_j \prod_{t=2}^{n-1} A_t + P_iP_j \prod_{t=1}^{n-1} A_t$$

Now using lemma, it is seen that the right hand side is always positive and thus,

$$\pi_i \pi_j - \pi_{ij} \geq 0$$

Hence the theorem,

In sampling scheme-1, some of the units may be repeated in the sample. Thus, the disadvantage of the sampling scheme-1 is that d, the number of distinct unts in the sample is a random variable. Clearly,

$$E(d) = 1 + \sum_{r=1}^{n-1} K_r \leq n$$

We shall denote E(d) by M throughout.

We shall now consider a sampling scheme for fixed sample size.

Sampling Scheme-2: This sampling scheme consists of the following steps.

(a) Select a sample of n units out of N units using sampling scheme-1. Let d, be the number of distinct units in the sample so selected.

(b) Select (n–d) units out of the remaining (N–d) units of the population by SRSWOR. Supplement it to the sample selected at step (a).

The inclusion probabilities for the individual and pairwise units for sampling scheme-2 are given by the following theorems.

Theorem-4 : For sampling scheme-2, the inclusion probability for the i-th unit is given by

$$\pi_i = E\left(\frac{n-d}{N-d}\right) + mP_i\left[1 - E\left(\frac{n-d}{N-d}\right)\right]$$

Proof: A unit is included in the sample if it is selected either at step (a) or at step (b). The probability of i-th unit being included in the sample at step (a) is mP_i and at step (b) is

$$(1-mP_i)\ E\left(\frac{n-d}{N-d}\right)$$

Therefore,

$$\pi_i = mP_i + (1-mP_i)\ E\left(\frac{n-d}{N-d}\right)$$

$$= E\left(\frac{n-d}{N-d}\right) + mP_i\left[1-E\left(\frac{n-d}{N-d}\right)\right]$$

Hence the theorem.

Theorem-5 : For sampling scheme-2, the inclusion probability for the i-th and j-th units together in the sample is given by

$$\pi_{ij} = \pi'_{ij}\left[1-E\left(\frac{n-d}{N-d}\right)\left\{2-E\left(\frac{n-d-1}{N-d-1}\right)\right\}\right] + E\left(\frac{n-d}{N-d}\right)$$
$$\left[1-E\left(\frac{n-d-1}{N-d-1}\right)\right](\pi'_i+\pi'_j) + E\left(\frac{n-d}{N-d}\right)E\left(\frac{n-d-1}{N-d-1}\right)$$

Where π'_i and π'_{ij} are the inclusion probabilities of the i-th unit and (i, j)-th pair of units respectively in the sample at step (a).

Proof : The inclusion probability for the pair of units (i, j) for sampling scheme-2 is given by

π_{ij} = Probability of selecting (i, j)-th pair of units at step (a) + [Probability of selecting i-th unit at step (a)] [Probability of selecting j-th unit at step (b)] + [Probability of selecting j-th unit at step (a)] [Probability of selecting i-th unit at step (b)] + [Probability of not selecting both i-th or j-th unit at step (a)] [Probability of selecting (i, j)-th pair of units at step (b)].

$$= \pi'_{ij} + (\pi'_i - \pi'_{ij})\, E\left(\frac{n-d}{N-d}\right) + (\pi'_j - \pi'_{ij})\, E\left(\frac{n-d}{N-d}\right)$$

$$+ (1 - \pi'_i - \pi'_j + \pi'_{ij})\, E\left(\frac{n-d}{N-d}\right) E\left(\frac{n-d-1}{N-d-1}\right)$$

$$= \pi'_{ij}\left[1 - E\left(\frac{n-d}{N-d}\right)\left\{2 - E\left(\frac{n-d-1}{N-d-1}\right)\right\}\right] + E\left(\frac{n-d}{N-d}\right)$$

$$\left[1 - E\left(\frac{n-d-1}{N-d-1}\right)\right](\pi'_i + \pi'_j) + E\left(\frac{n-d}{N-d}\right) E\left(\frac{n-d-1}{N-d-1}\right)$$

Hence the theorem.

Since N is sufficiently large as compared to d, for simplicity of expressions it is reasonable to assume here that

$$E\left(\frac{1}{N-d}\right) \approx \frac{1}{N - E(d)}$$

Under this assumption π'_is and π'_{ij}s simplify to

$$\pi_i = \frac{n-m}{N-m} + \frac{N-n}{N-m}\,\pi'_i$$

and

$$\pi_{ij} = \pi'_{ij}\frac{(N-n)(N-n-1)}{(N-m)(N-m-1)} + \frac{(N-n)(n-m)}{(N-m)(N-m-1)}(\pi'_i + \pi'_j)$$

$$+ \frac{(n-m)(n-m-1)}{(N-m)(N-m-1)}$$

If π'_is are desired to be proportional to size in sampling scheme-2, we can revise the probabilities from P_i to P'_i at step (a) such that the resulting sampling scheme is a πPS sampling scheme. In that

$$\pi_i = m'P'_i + (1 - m'P'_i)\, E\left(\frac{n-d}{N-d}\right).$$

Where m' is E(d) based on revised probabilities

$$\text{or } nP_i = E\left(\frac{n-d}{N-d}\right) + m'P'_i\left[1 - E\left(\frac{n-d}{N-d}\right)\right]$$

This gives

$$P_1' = \frac{nP_1 - E\left(\frac{n-d}{N-d}\right)}{m' - \left[1 - E\left(\frac{n-d}{N-d}\right)\right]}.$$

It may be observed that m′ and $E\left(\frac{n-d}{N-d}\right)$ depend upon P_1 and therefore the solution of P_1 is not explicit. However, an approximate solution of P_1 can be obtained by substituting in the right hand side the quantities based on the initial probabilities of selection. The solution so obtained is

$$P_1' = \frac{nP_1(N-m)-(n-m)}{(N-n)\,m} \tag{1}$$

Since P_i''s are all positive we should have

$$P_1 > \frac{n-m}{n(N-m)}$$

This imposes a restriction. However, this restriction is not at all severe.

For this solution of revised probabilities of selection at step (a) in sampling scheme-2, the expression for π_1 can be seen to be equal to

$$\pi_1 = m'P_1' + (1-m'P_1')\left(\frac{n-m'}{N-m'}\right).$$

Substituting the value of P_1' from equation (1) in π_1, we get

$$\pi_1 = \frac{m'}{m}\,\frac{N-m}{N-m'}\,nP_1 + \frac{n-m'}{N-m'}$$

Since for large N :

$$\frac{1-\frac{m'}{N}}{1-\frac{m}{N}} \approx 1, \quad \frac{m'}{m} \text{ is of order one and } \frac{m-m'}{N-m'} \text{ is of order zero,}$$

and we get $\pi_1 \approx n\,P_1$

Therefore, sampling scheme-2 with revised probabilities as in (1) is nearly πPS sampling scheme.

Illustration : Consider a population of size 13 with X value as :

2, 2, 3, 3, 4, 4, 5, 6, 6, 7, 7, 8, 8

For this population K_r's obtained were

$K_1 = .90893$, $K_2 = .82427$ and $K_3 = .74557$

Making use of these K_r's the revised probabilities of selection to be used at step (a) for different units were obtained which are being presented in column (3) of table 1. The inclusion probabilities of different units under nearly πPS sampling scheme and the desired inclusion probabilities are being presented in columns (4) and (5) respectively of table 1.

Table 1. Revised probabilities and inclusion probabilities under nearly πPS sampling scheme.

Unit No.	Original probabilities P_i	Revised probabilities P_i'	Calculated inclusion probabilities π_i	Desired inclusion probabilities $\pi_i = nP_i$
1	.0307692	.0207803	.1257671	.1230768
2	.0307692	.0207803	.1257671	.1230768
3	.0461538	.0394945	.1864087	.1846152
4	.0461538	.0394945	.1864087	.1846152
5	.0615384	.0582087	.2470502	.2461536
6	.0615384	.0582087	.2470502	.2461536
7	.0769230	.0769229	.3054431	.3076920
8	.0923076	.0956372	.3683335	.3692304
9	.0923076	.0956372	.3683335	.3692304
10	.1076923	.1143515	.4289753	.4307692
11	.1076923	.1143515	.4289753	.4307692
12	.1230769	.1330657	.4896168	.4923076
13	.1230769	.1330657	.4896168	.4923076

Illustration 2 : Consider a population of size 15 with X values as

X : 13, 13, 13, 14, 14, 14, 15, 15, 15, 16, 16, 16, 17, 17, 17.

For this population K_r's obtained were

$$K_1 = .9327, \quad K_2 = .8698 \text{ and} \qquad K_3 = .8111$$

Making use of these K_r's the revised probabilities of selection to be used at step (a) for different units were obtained which are being presented in column (3) of table-2. The inclusion probabilities of different units under nearly πPS sampling scheme and the desired inclusion probabilities are being presented in columns (4) and (5) respectively of table-2.

Table 2. Revised probabilities and inclusion probabilities under nearly πPS sampling scheme.

Unit No.	Original probabilities P_i	Revised probabilities P_i'	Calculated inclusion probabilities π_i	Desired inclusion probabilities $\pi_i = nP_i$
1	.0577777	.0564818	.2311260	.2259272
2	.0577777	.0564818	.2311260	.2259272
3	.0577777	.0564818	.2311260	.2259272
4	.0622222	.0615742	.2488962	.2462968
5	.0622222	.0615742	.2488962	.2462968
6	.0622222	.0615742	.2488962	.2462968
7	.0666666	.0666665	.2666666	.2666666
8	.0666666	.0666665	.2666666	.2666666
9	.0666666	.0666665	.2666666	.2666666
10	.0711111	.0717590	.2844365	.2870360
11	.0711111	.0717590	.2844365	.2870360
12	.0711111	.0717590	.2844365	.2870360
13	.0755555	.0768513	.3022063	.3074852
14	.0755555	.0768513	.3022063	.3074852
15	.0755555	.0768513	.3022063	.3074852

It can be seen that the proposed sampling scheme provides inclusion probabilities very near to the desired value.

The expression for π_{ij} under sampling scheme-2 with revised probabilities is given by

$$\pi_{ij} = \frac{N-n}{N-m'} m' (P_i' + P_j') - \frac{(N-n)(N-n-1)}{(N-m')(N-m'-1)} \Bigg[1 - (1 - P_i' - P_j') \left(1 - \frac{K_1 P_i'}{1-P_i'} - \frac{K_1 P_j'}{1-P_j'}\right) \cdots \left\{ 1 - \frac{K_{n-1}' P_i'}{1-(1+K_1'+\ldots+K_{n-2}')P_i'} - \frac{K_{n-1}' P_j'}{1-(1+K_1'+\ldots+K_{n-2}')P_j'} \right\} \Bigg] + \frac{(n-m')(n-m'-1)}{(N-m')(N-m'-1)}$$

Theorem 6: For the nearly πPS sampling scheme-2, Yates-Grundy form of variance estimator is always non-negative.

Proof: The condition of Yates-Grundy variance estimator to be non-negative is

$$\pi_i \pi_j - \pi_{ij} \geq 0 \text{ for all } i \neq j = 1, 2, \ldots, N$$

It is known that

$$\pi_i = \frac{n-m'}{N-m'} + \frac{N-n}{N-m'} \pi_i'$$

and

$$\pi_{ij} = \frac{(N-n)(N-n-1)}{(N-m')(N-m'-1)} \pi_{ij}' + \frac{(N-n)(N-m')}{(N-m')(N-m'-1)} (\pi_i' + \pi_j') + \frac{(n-m')(n-m'-1)}{(N-m')(N-m'-1)}$$

Then

$$\pi_i \pi_j - \pi_{ij} = \left[\frac{n-m'}{N-m'} + \frac{N-n}{N-m'} \pi_i'\right] \left[\frac{n-m'}{N-m'} + \frac{N-n}{N-m'} \pi_j'\right] - \frac{(n-m')(n-m'-1)}{(N-m')(N-m'-1)} - \frac{(N-n)(n-m')}{(N-m')(N-m'-1)} (\pi_i' + \pi_j') - \frac{(N-n)(N-n-1)}{(N-m')(N-m'-1)} \pi_{ij}' - \left(\frac{N-n}{N-m'}\right)^2 (\pi_i' \pi_j' - \pi_{ij}')$$

$$+ \frac{(n-m')(N-n)}{(N-m')^2\ (N-m'-1)}\ (1-\pi'_i-\pi'_j+\pi'_{ij})$$

Since $\pi'_i\pi'_j-\pi'_{ij}\geq 0$ from the result of Theorem 3 and $(1-\pi'_i-\pi'_j+\pi'_{ij})>0$ because this is the probability of not selecting both i-th or j-th unit at step (a) of sampling scheme-2, we have

$$\pi_i\pi_j-\pi_{ij}\geq 0$$

Hence the theorem.

3. *Numberical Comparison*

It is very difficult to give a theoretical comparison of the proposed nearly PS sampling scheme with the known πPS sampling schemes. However, we consider the following examples for studying the performance of nearly πPS sampling scheme as compared to some of the known πPS sampling schemes for n = 4.

Example-1: Three populations of size 13 each have been generated. The size measure for these three populations is same but the studying variate generated has the following properties for the

Table 3. Populations considered in the example

Unit No.	Six measure X	Character under study(y)		
		Population–1	Population–2	Population–3
1	2	16.0	28.0	16.0
2	2	17.0	27.0	17.0
3	3	27.0	39.0	27.0
4	3	28.5	37.5	28.5
5	4	40.0	48.0	40.0
6	4	42.0	46.0	42.0
7	5	55.0	55.0	55.0
8	6	69.0	63.0	63.0
9	6	72.0	60.0	60.0
10	7	87.0	66.5	66.5
11	7	91.0	63.0	63.0
12	8	108.0	68.0	68.0
13	8	112.0	64.0	64.0

population 1, 2 and 3 respectively. For population 1, Y/X and X have perfect positive correlation, for population 2, Y/X has perfect negative correlation with X whereas for population 3, Y/X and X are uncorrelated. These populations are similar to those considered by Cochran [1] and are being presented in Table 3.

Example-2 : Three more population of size 15 each have been generated satisfying the finite population g-model as under:

$$Y = \beta X + \varepsilon \quad \text{with} \quad E\left(\frac{\varepsilon}{X}\right) = 0$$

$$\text{and} \quad E\left(\frac{\varepsilon^2}{X}\right) = AX^g$$

The population so generated with the values of g as 0, 1 and 2 are being presented in table 4.

Table 4. Populations considered in the example–2

Unit No.	Six measure X	Character under study(Y)		
		Population–4 g = 0	Population–5 g = 1	Population–6 g = 2
1	13	25.00	22.34	13.00
2	13	26.00	26.00	26.00
3	13	27.00	29.66	39.00
4	14	27.00	24.26	14.00
5	14	28.00	28.00	28.00
6	14	29.00	31.74	42.00
7	15	29.00	26.13	15.00
8	15	30.00	30.00	30.00
9	15	31.00	33.87	45.00
10	16	31.00	28.00	16.00
11	16	32.00	32.00	32.00
12	16	33.00	36.00	48.00
13	17	33.00	29.88	17.00
14	17	34.00	34.00	34.00
15	17	35.00	38.12	51.00

The sampling schemes considered for comparison are

(a) Nearly πPS sampling scheme

(b) Sampford's πPS sampling scheme

(c) Randomised PPS systematic sampling

(d) PPS with replacement sampling

The variance of the estimates for n = 4 under different sampling schemes and for various populations are presented in table-5.

Table 5. Variance of different strategies for different populations

Populations	Sampling schemes			
	(a)	(b)	(c)	(d)
1	1994.50	2381.17	2354.61	3060.99
2	1682.26	2376.74	2349.94	2354.93
3	586.11	687.93	759.97	987.97
4	26.82	30.33	71.70	89.63
5	408.41	452.19	494.13	617.66
6	6436.61	6730.67	6798.32	8497.90

It can be seen from these results that the performance of the proposed sampling scheme is highly satisfactory and the gain in efficiency over the existing sampling schemes is quite considerable.

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